Technology Brief 5: Global Positioning System

The Global Positioning System (GPS), initially developed in the 1980s by the U.S. Department of Defense as a navigation tool for military use, has evolved into a system with numerous civilian applications, including vehicle tracking, aircraft navigation, map displays in automobiles and hand-held cell phones (Fig. TF5-1), and topographic mapping. The overall GPS comprises three segments. The space segment consists of 24 satellites (Fig. TF5-2), each circling Earth every 12 hours at an orbital altitude of about 12,000 miles and transmitting continuous coded time signals. All satellite transmitters broadcast coded messages at two specific frequencies: 1.57542 GHz and 1.22760 GHz. The user segment consists of hand-held or vehicle-mounted receivers that determine their own locations by receiving and processing multiple satellite signals. The third segment is a network of five ground stations, distributed around the world, that monitor the satellites and provide them with updates on their precise orbital information.

▶ GPS provides a location inaccuracy of about 30 m, both horizontally and vertically, but it can be improved to within 1 m by differential GPS. (See final section.) ◄

Principle of Operation

The triangulation technique allows the determination of the location \((x_0, y_0, z_0)\) of any object in 3-D space from knowledge of the distances \(d_1, d_2, \text{ and } d_3\) between that object and three other independent points in space of known locations \((x_1, y_1, z_1)\) to \((x_3, y_3, z_3)\). In GPS, the distances are established by measuring the times it takes the signals to travel from the satellites to the GPS receivers, and then multiplying them by the speed of light \(c = 3 \times 10^8 \text{ m/s}\). Time synchronization is achieved by using atomic clocks. The satellites use very precise clocks, accurate to 3 nanoseconds \((3 \times 10^{-9} \text{ s})\), but receivers use less accurate, inexpensive, ordinary quartz clocks. Consequently, the receiver clock may have an unknown time offset error \(t_0\) relative to the satellite clocks. To correct for the time error of a GPS receiver, a signal from a fourth satellite is needed.

The GPS receiver of the automobile in Fig. TF5-3 is at distances \(d_1\) to \(d_4\) from the GPS satellites. Each satellite sends a message identifying its orbital coordinates \((x_1, y_1, z_1)\) for satellite 1, and so on for the other satellites, together with a binary-coded sequence common to all satellites. The GPS receiver generates the same binary sequence (Fig. TF5-3),

Figure TF5-1 iPhone map feature.

Figure TF5-2 GPS nominal satellite constellation. Four satellites in each plane, 20,200 km altitudes, 55° inclination.
and by comparing its code with the one received from satellite 1, it determines the time $t_1$ corresponding to travel time over the distance $d_1$. A similar process applies to satellites 2 to 4, leading to four equations:

\[
\begin{align*}
    d_1^2 &= (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 = c [(t_1 + t_0)]^2 \\
    d_2^2 &= (x_2 - x_0)^2 + (y_2 - y_0)^2 + (z_2 - z_0)^2 = c [(t_2 + t_0)]^2 \\
    d_3^2 &= (x_3 - x_0)^2 + (y_3 - y_0)^2 + (z_3 - z_0)^2 = c [(t_3 + t_0)]^2 \\
    d_4^2 &= (x_4 - x_0)^2 + (y_4 - y_0)^2 + (z_4 - z_0)^2 = c [(t_4 + t_0)]^2.
\end{align*}
\]

The four satellites report their coordinates $(x_1, y_1, z_1)$ to $(x_4, y_4, z_4)$ to the GPS receiver, and the time delays $t_1$ to $t_4$ are measured directly by the receiver. The unknowns are $(x_0, y_0, z_0)$, the coordinates of the GPS receiver, and the time offset of its clock $t_0$. Simultaneous solution of the four equations provides the desired location information.

Differential GPS

The 30 m GPS position inaccuracy is attributed to several factors, including time-delay errors (due to the difference between the speed of light and the actual signal speed in the troposphere) that depend on the receiver’s location on Earth, delays due to signal reflections by tall buildings, and satellites’ locations misreporting errors.

▶ **Differential GPS**, or DGPS, uses a stationary reference receiver at a location with known coordinates. ◀

By calculating the difference between its location on the basis of the GPS estimate and its true location, the reference receiver establishes coordinate correction factors and transmits them to all DGPS receivers in the area. Application of the correction information usually reduces the location inaccuracy down to about 1 m.